2nd MEMO, Olomouc, Czech Republic

Individual competition, September 6, 2008

I–1: Let $(a_n)_{n=1}^{\infty}$ be any increasing sequence of positive integers with the following property: for each quadruple of indices (i, j, k, m), where $1 \leq i < j \leq k < m$ and $i + m = j + k$, the inequality $a_i + a_m > a_j + a_k$ holds. Determine the least possible value of a_{2008}.

I–2: Consider a $n \times n$ chessboard, where $n > 1$ is an integer. In how many ways can we put $2n - 2$ identical stones on the chessboard (each on another square) such that no two stones lie on the same diagonal? (By a diagonal we mean a row of squares whose diagonals of one direction lie on the same line).

I–3: Let ABC be an isosceles triangle with $|AC| = |BC|$. Its incircle touches AB and BC at D and E, respectively. A line (different from AE) passes through A and intersects the incircle at F and G. The lines EF and EG intersect the line AB at K and L, respectively. Prove that $|DK| = |DL|$.

I–4: Find all integers k such that for every integer n, the numbers $4n + 1$ and $kn + 1$ are relatively prime.

Each problem is worth 8 points.

The order of the problems does not depend on their difficulty.

Time: 5 hours

Time for questions: 45 min