1. MEMO, Eisenstadt, Austria

Team competition, September 23, 2007

5. Let a, b, c, d be arbitrary real numbers from the closed interval $[\frac{1}{2}, 2]$ satisfying $abcd = 1$. Find the maximal value of

$$\left(a + \frac{1}{b}\right)\left(b + \frac{1}{c}\right)\left(c + \frac{1}{d}\right)\left(d + \frac{1}{a}\right).$$

6. For a set P of five points in the plane in general position, we denote the number of acute-angled triangles with vertices in P by $a(P)$ (a set of points is said to be in general position if no three points lie on a line). Determine the maximal value of $a(P)$ over all possible sets P.

7. Let $s(T)$ denote the sum of the lengths of the edges of a tetrahedron T. We consider tetrahedra with the property that the six lengths of their edges are pairwise different positive integers, where one of them is 2 and and another one of them is 3.

   1. Find all positive integers n for which there exists a tetrahedron T with $s(T) = n$.
   2. How many such pairwise non congruent tetrahedra T with $s(T) = 2007$ exist?

   Two tetrahedra are said to be non congruent, if one cannot be transformed by reflections with respect to planes, translations and/or rotations into the other.

   (It is not necessary to prove that the tetrahedra are not degenerated i.e. have positive volume.)

8. Determine all positive integers k with the following property: there exists an integer a such that $(a + k)^3 - a^3$ is a multiple of 2007.

Each problem is worth 8 points.

The order of the problems does not depend on their difficulty.

Time: 5 hours

Time for questions: 45 min